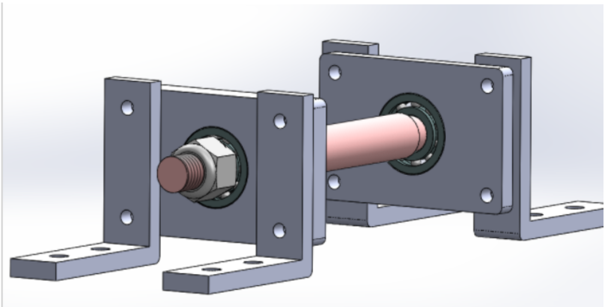
Rotational Motion Module Design

This design script serves to predict the load capacity and stiffness of the following rotational motion module based on the bearings / bushings used. It was also used in the evaluation of the appropriate interference fits for fitting the bearings into their housing.



Material Properties

```
potential_materials = [ 'Brass', 'Nylon' 'Steel', 'Aluminium' ];
yield_stress = [ 135E6, 45E6, 435E6, 276E6 ];        % [Pa]
elastic_modulus = [ 105E9, 2.6E9, 200E9, 69E9 ];     % [Pa]
poisson_ratio = [ 0.346, 0.39, 0.26, 0.33 ];
ultimate_strength = [ 345E6, 82.7E6, 670E6, 310E6 ]; % [Pa]
```

Input Parameters

The input parameters were obtained from the technical specifications for the bearings and the bushing used. The two bearings were Koyo EE4C3 deep groove ball bearings and the bushing was McMaster #6338K424: all parts were found in the Gelb lab.

```
OD_bearing = 28.575E-3;             % [m]
C_r = 5107;                         % [N], dynamic load rating
C_0r = 2402;                        % [N], static load rating
vmax_bearing = 27000;               % [RPM], maximum rated speed
D_ball = 3.54E-3;                   % [m]
bearing_thickness = 6.345E-3;       % [m]
shim_thickness = 0.1E-3;            % [m]
L_shaft = 0.2;                      % [m]
D_shaft = 12.7E-3;                  % [m]
I_shaft = pi * D_shaft^4 / 64;      % [m]
% Shaft Material Choice: 1 = Brass, 2 = Nylon, 3 = Steel, 4 = Aluminium
shaft_material_choice = 4;
E_shaft = elastic_modulus( shaft_material_choice ); % [Pa]
shaft_straightness = 2E-4;         % [m], from McMaster's technical specifications.
P = 500;                           % [N], normal operating load of spindle (just a guess for
```

Dynamic Load Capacity

```
max_dynamic_load = 3 * C_r;
```

```
disp('The maximum total dynamic load capacity of the rotational module is ' + string( round( r
```

```
The maximum total dynamic load capacity of the rotational module is 15300 Newtons.
```

Static Load Capacity

```
max_static_load = 3 * C_0r;
disp('The maximum total static load capacity of the rotational module is ' + string( round( ma
```

```
The maximum total static load capacity of the rotational module is 7210 Newtons.
```

Radial Stiffness

$$k_{\text{radial}} = \frac{F_{\text{radial}}}{0.01 * D_{\text{ball}}}$$

```
k_radial_bearing = C_r / ( 0.01 * D_ball );
k_radial_fixed = 1 / ( 2 / k_radial_bearing );
k_radial_total = 1 / ( 3 / k_radial_bearing );
disp('The approximate radial stiffness of the rotational module is ' + string( round( k_radia
```

```
The approximate radial stiffness of the rotational module is 48100000 Newtons per meter.
```

Roll Stiffness of Fixed End

$$k_{\text{axial}} = k_{\text{radial set}} * \left(\frac{\text{Bearing Spacing}}{2}\right)^2$$

```
k_axial_set = k_radial_fixed * (( bearing_thickness + shim_thickness ) / 2 )^2;
disp('The axial stiffness of the fixed end set of bearings is ' + string( round( k_axial_set,
```

```
The axial stiffness of the fixed end set of bearings is 749 Newtons per meter.
```

Expected Life of Bearings

$$L_{10} = \left(\frac{C_0}{P}\right)^3$$

This calculates the expected number of revolutions the bearing, after which 10% of a set of bearings fail.

```
L_10 = ( C_r *3/ P )^3;
disp('The estimated life of the bearings is ' + string( round( L_10, 3, 'significant' )) + ' r
```

```
The estimated life of the bearings is 28800 revolutions.
```

Shaft Stiffness

I am going to average the results for the shaft bending between a fixed-fixed shaft and a simply supported shaft.

$$\delta_{\text{simply supported max}} = \frac{5PL^3}{384EI}$$

$$\delta_{\text{fixed fixed}}(x) = \frac{PL^3}{384EI}$$

$$\delta_{\text{shaft}} = \frac{\delta_{\text{simply supported}} + \delta_{\text{fixed fixed}}}{2}$$

```
k_simply_supported = 384 * E_shaft * I_shaft / ( 5 * L_shaft^3 );
k_fixed_fixed = 384 * E_shaft * I_shaft / L_shaft^3;
k_shaft = ( k_simply_supported + k_fixed_fixed ) / 2;
disp('The stiffness of the shaft is ' + string( round( k_shaft, 3, 'significant' )) + ' Newtor
```

```
The stiffness of the shaft is 2540000 Newtons per meter.
```

Accuracy

```
accuracy = rad2deg( asin( shaft_straightness / L_shaft ));
disp('The angular accuracy is ' + string( round( accuracy, 3, 'significant' )) + ' degrees.')
```

```
The angular accuracy is 0.0573 degrees.
```

Repeatability

The repeatability was estimated to be three times better than the accuracy.

```
repeatability = accuracy / 3;
disp('The angular repeatability is ' + string( round( repeatability, 3, 'significant' )) + ' c
```

```
The angular repeatability is 0.0191 degrees.
```

Speed Limit

```
disp('The maximum rated bearing speed is ' + string( round( vmax_bearing, 3, 'significant' ))
```

```
The maximum rated bearing speed is 27000 RPM.
```